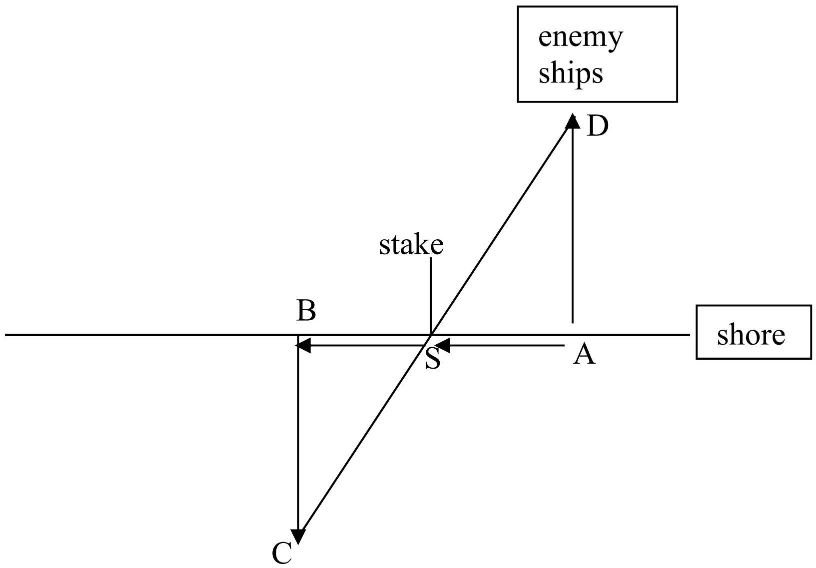
An ancient tale relates the following: The king of an island domain saw enemy ships on the horizon. He wondered how far away these ships were, because he wanted to know how long he had to prepare before they attacked. He asked a wise mathematician in his court if he could determine the ships' distance from shore. The mathematician carried out the following measurements:



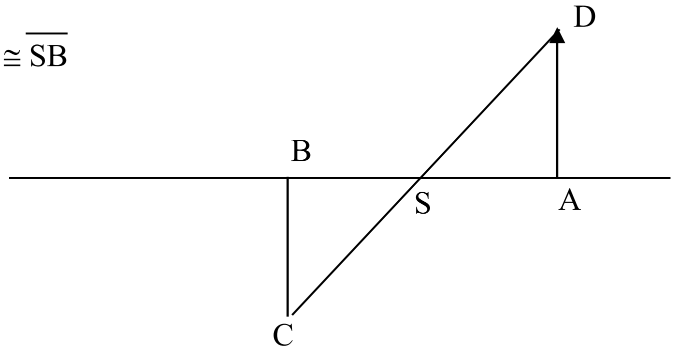
He started at point A, where the shore is perpendicular to his line-of-sight to the ships.
He walked along the shore 100 paces, and placed a stake at point S.
He continued walking along the shore another 100 paces, to point B.
He then turned 90 degrees and walked directly *away* from the shore.

As he walked, he kept checking to see if the stake at S was in line with the ships.
When the stake lined up with the ships, he was at point C.
He now announced to the King that he could answer the King's original request. *Why?*

By carrying out this procedure, the mathematician formed a pair of congruent triangles: specifically $\Delta ADS \cong \Delta BCS$. Be sure you see why this is before going on.
Thus, the distance the King wanted to know, AD, is the same as the distance BC that the mathematician had just traveled.

A formal statement-reason proof of this result might look like the following:

Given: $\overline{AD} \perp \overline{AB}$, $\overline{BC} \perp \overline{AB}$, $\overline{AS} \cong \overline{SB}$
Prove: $\overline{BC} \cong \overline{AD}$

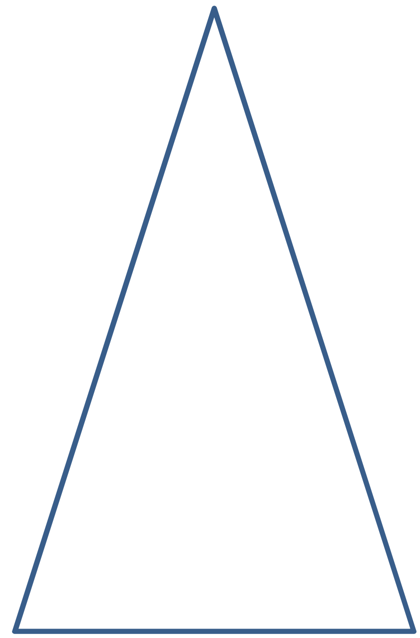


Statement	Reason
1. $\overline{AD} \perp \overline{AB}$, $\overline{BC} \perp \overline{AB}$, $\overline{AS} \cong \overline{SB}$	1. given
2. $\angle DAS$ and $\angle CBS$ are right	2. def'n $\perp$
3. $\angle DAS \cong \angle CBS$	3. All right <'s are $\cong$
4. $\angle ASD \cong \angle BSC$	4. Vertical <'s are $\cong$
5. $\Delta ADS \cong \Delta BCS$	5. ASA
6. $\overline{BC} \cong \overline{AD}$	**6. Def'n of Congruent Triangles, or Corresponding parts of congruent triangles are congruent (may be abbreviated CPCTC)**

Notice that we have used one of our familiar triangle congruence properties--ASA--but not just for the sake of proving two triangles congruent. It was a stepping stone to our final result: proving the two segments $\overline{BC}$ and $\overline{AD}$ are congruent.

Reason #6, printed in bold above, allows us to conclude that a pair of corresponding parts (sides or angles) are congruent. Notice that we need to have proved a pair of congruent triangles *before* we can use "CPCTC."

Isosceles Triangle Theorem – Part 1



Consider the isosceles triangle pictured at right.
What is the definition of an isosceles triangle?

A couple more definitions:

Vertex angle: the angle between the two congruent sides
Base angles: the angles opposite the two congruent sides

Draw an example of an isosceles triangle on paper.
Fold it in half so the congruent sides match up.
What else seems to be congruent?

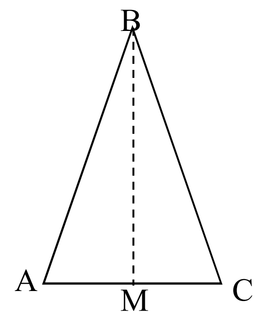
Write a conjecture:

Justify your conjecture using a triangle congruence property. (Hint: When you folded your paper triangle in half you created two triangles. Decide how to define the line where you folded. If you first choice of definition leads to a dead end, try a different definition.)

Write a formal deductive argument proving your conjecture.

Did your deductive argument look something like the following?
(There are multiple ways to correctly prove this – and most other – facts.)

The base angles of an isosceles triangle are congruent.



Given: $\overline{AB} \cong \overline{BC}$
Prove: $\angle A \cong \angle C$

Statement	Reason
1. $\overline{AB} \cong \overline{BC}$	1. given
2. Draw M, midpoint of $\overline{AC}$	2. A segment has exactly one midpt.
3. Draw $\overline{BM}$	3. Two points determine a line.
4. $\overline{AM} \cong \overline{MC}$	4. def'n midpoint
5. $\overline{MB} \cong \overline{MB}$	5. reflexive property
6. $\Delta ABM \cong \Delta CBM$	6. SSS
7. $\angle A \cong \angle C$	7. CPCTC

Isosceles Triangle Theorem – Part 2

Sometimes mathematicians look at the converses of known facts in their search for possible new theorems.

Consider the converse of the theorem we just proved.

Conjecture:

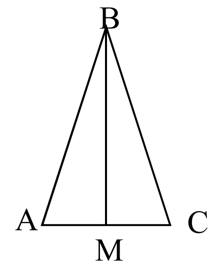
Does it seem plausible?

Justify your conjecture with a formal two-column proof:

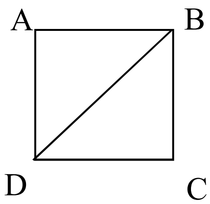
Summarize: Write the results of Isosceles Triangle Theorem Part 1 and Part 2 as an "if and only if" statement. Add it to your Theorem List. Now this fact can be used in later proofs.

More Practice Proofs

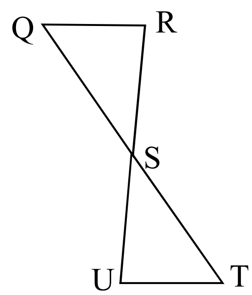
1. In an isosceles triangle, the segment that bisects the vertex angle also bisects the base.
   Given: $\overline{AB} \cong \overline{CB}$
   $\overline{BM}$ bisects $\angle$ABC
   Prove: $\overline{AM} \cong \overline{MC}$



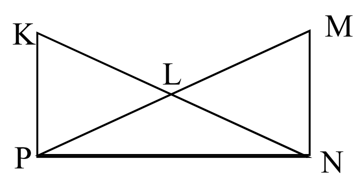
2. Given: ABCD is a square.
   Prove: $\overline{AB} \parallel \overline{CD}$



3. Given: S is the midpoint of both $\overline{RU}$ and $\overline{QT}$
   Prove: $\overline{QR} \cong \overline{UT}$



4. Given: $\overline{KP} \perp \overline{PN}$; $\overline{MN} \perp \overline{PN}$.
   $\angle$LPN $\cong$ $\angle$LNP
   Prove: $\overline{KP} \cong \overline{MN}$



5. Given: $\overline{EH} \cong \overline{EF}$
   $\angle$HEI $\cong$ $\angle$FEI
   Prove: $\overline{EI} \perp \overline{HF}$